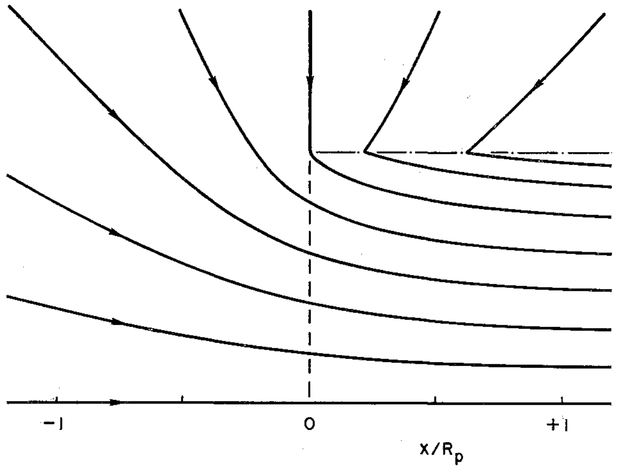

Fig. 3 Perturbation mean flow streamlines for constant blade circulation.

find that for radii greater than R_p, the streamlines are exactly symmetric about the propeller plane. By continuity, this substantiates the doubling of the slipstream velocity at infinity as found from a momentum balance. The streamline passing through the propeller tip approaches with an infinite slope and then contracts far downstream to a radius of 70.7% of the propeller radius, as can be easily verified from the continuity equation. Downstream of the propeller plane and outside the tip streamline, the flow approaches the propeller and then abruptly is swept back downstream as it first crosses the vortex system trailing from the blades. Perhaps the most striking feature that we notice, though, is the appearance of a "natural hub" created within the flowfield. This is caused by the local inner trailing vortex system. These vortices are of opposite sign to those further outboard and so induce a flow "upstream." As a result, we expect that the hub for a real propeller probably does not disturb the flow as much as might be thought beforehand. In Fig. 1, this natural hub effect has almost been masked out by the freestream contribution, which dominates the flowfield.

Figure 3 shows the results for the perturbation streamlines for the case of constant circulation distribution. This can be related to the streamline pattern for a circular sink disk of uniform strength; see for example Ref. 4. Comparing the results of Fig. 3 with those of Fig. 2, we find that the extent of the influence of the nonuniformity of the circulation is very local and appears only in the immediate vicinity of the propeller itself. Again, the streamlines are exactly symmetric about the propeller plane outside the slipstream and the tip streamline contracts to a value of 70.7% of the propeller radius as before.

For the static condition, we have been able to correlate certain features of these perturbation streamlines, e.g., the rapid turning of the flow near the propeller tip and the overall slipstream contraction, with experimental smoke-visualization pictures. Other features that do not correlate serve to remind us of the basic deficiency of the classical vortex model, which assumes that the wake is made up of regular helices and does not contract.[5] Nevertheless, it is very useful at least insofar as a simple picture of the static flowfield is concerned.

Conclusions

Results have been presented which show the effect of the shape of the propeller blade circulation distribution upon the mean flow streamlines. This effect is found to be confined to the very immediate vicinity of the propeller itself, and decreases quite rapidly as the forward flight velocity increases. For the perturbation streamlines, a natural hub is observed in the flowfield for the representative circulation and so the real propeller hub probably does not disturb the flow as much as might be thought.

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Effect of Back Pressure on Nozzle Thrust

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Nomenclature

A = streamtube flow area
F = thrust
m = mass flow rate
M = Mach number
P = total pressure
p = static pressure
V = velocity
γ = ratio of specific heats

Subscripts

e = exit plane, actual conditions
i = exit plane, ideal conditions at pressure p_0
0 = ambient atmosphere
t = upstream of nozzle, stagnation conditions

IT is well known that the thrust of a supersonic nozzle is a maximum when the nozzle exit static pressure p_e is equal to the ambient atmospheric pressure p_0. Experience indicates this is not true for sonic or subsonic nozzles, especially when operating at low nozzle pressure ratios. In this note we shall examine the variation of nozzle thrust when the exit static pressure differs from ambient pressure. This case has received little analytical attention in the past, although there are many practical situations in which such variations can occur. Curvature in an exhaust duct, for example, can produce pressure gradients at the nozzle exit plane,[1] which permits some of the flow to exhaust at a pressure lower than ambient. For small radii of curvature the pressure variation may be quite marked. In other cases the nozzle exit plane may be located in a pressure field that is above ambient pressure. Examples are a nozzle exhausting from a boattail body, or a nozzle discharging normal to a ground plane (ground-effect machine).

Consider a small streamtube in which the flow is approximately one-dimensional. For a constant mass flow m_e we

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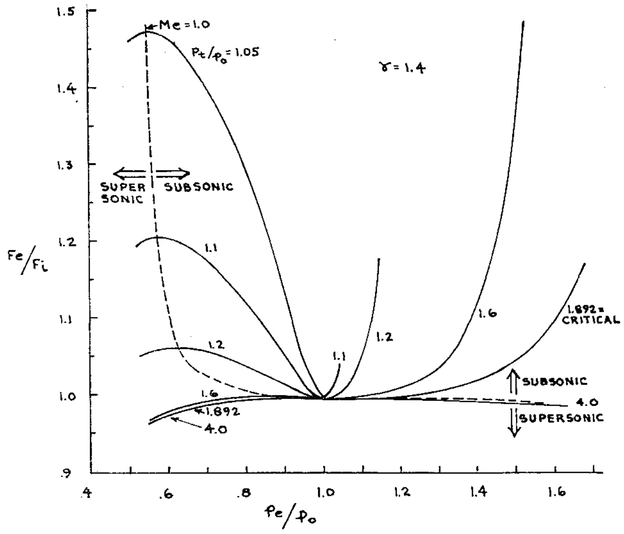

Fig. 1 Effect of nozzle exit pressure on thrust.

define the thrust ratio of actual to ideal thrust as

$$F_e/F_i = [m_eV_e + (p_e - p_0)A_e]/m_eV_i \tag{1}$$

For a perfect gas with constant specific heat ratio we find

$$F_e/F_i = (V_e/V_i)\{1 + [(p_e - p_0)/(p_e\gamma M_e^2)]\} \tag{2}$$

If we further assume isentropic flow without heat addition and use standard one-dimensional relationships,[2] for example,

$$P_t/p = \{1 + [(\gamma - 1)/2]M^2\}^{\gamma/(\gamma-1)} \tag{3}$$

We can express Eq. (2) in terms of the actual and ideal exit Mach number,

$$\frac{F_e}{F_i} = \frac{M_e}{M_i}\left(\frac{1 + \frac{\gamma - 1}{2}M_i^2}{1 + \frac{\gamma - 1}{2}M_e^2}\right)^{1/2} \times \left(1 + \frac{1 - \left\{\frac{1 + [(\gamma - 1)/2]M_i^2}{1 + [(\gamma - 1)/2]M_e^2}\right\}^{\gamma/(1-\gamma)}}{\gamma M_e^2}\right) \tag{4}$$

where M_i is the exit Mach number for expansion to ambient pressure and M_e is for expansion to the actual exhaust pressure. It should be noted that the exit area for subsonic flow must be varied to maintain the same mass flow.

From Eq. (4), F_e/F_i may be plotted as a function of M_e and M_i, but a more convenient presentation is in terms of the nozzle pressure ratio (P_t/p_0) and the exit pressure ratio (P_e/p_0). This is shown in Fig. 1 with a dashed line to illustrate the regions of subsonic and supersonic flow at the nozzle exit. These results reveal some interesting trends. First we note that above or to the right of the dashed line the exit flow is subsonic. In this region, the thrust ratio increases continuously with exit pressure for nozzle pressure ratios (P_t/p_0) less than critical (1.892 in this case). This is typical of the ground-effect machine in which a very large thrust results with a low exit velocity. At higher nozzle pressure ratios, the thrust increase is less pronounced.

Secondly, we note that for exit pressures lower than atmospheric, the thrust of a subsonic flow again rises, reaching a peak value (at $M_e = 1$) for nozzle pressure ratios less than critical. For supersonic exit flow (to the left of dashed line), the thrust drops from the peak because the increase in exit velocity is more than offset by the lowered pressure.

The effect of exit pressure is most marked when the basic nozzle pressure ratio is small. Thus, for example, with a nozzle pressure ratio of $P_t/p_0 = 1.2$, a local exit pressure reduction of 40% or a pressure increase of 10% will each produce a thrust increase of about 6% above the ideal theoretical value.

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Rapid Estimation of Wing Aerodynamic Characteristics for Minimum Induced Drag

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Nomenclature

AR	= aspect ratio = (span)²/wing area
C_{LD}	= design lift coefficient
$C_{L\alpha}$	= lift coefficient per degree, ($dC_L/d\alpha_L$)
C_{mD}	= pitching moment about wing apex at design lift condition
C_{m_0}	= pitching moment at zero lift for $(\epsilon_T/\alpha)_{OPT}$ condition
$C_{m_{0_1}}$	= pitching moment coefficient about wing apex for $C_L = 0$ and $\epsilon_T = -1°$
$C_{m\alpha}$	= pitching moment coefficient about wing apex per degree, ($dC_m/d\alpha_L$)
M	= Mach number
α	= total angle of attack = ($\alpha_0 + \alpha_L$), deg
α_0	= zero-lift angle of attack for $(\epsilon_T/\alpha)_{OPT}$ condition, deg
α_{0_1}	= zero-lift angle of attack for 1° tip washout, deg
α_L	= flat-plate angle of attack needed to develop desired C_{LD}, deg
β	= Prandtl compressibility factor = $(1 - M^2)^{1/2}$
ϵ_T	= twist at tip (washout is negative), deg
$(\epsilon_T/\alpha)_{OPT}$	= tip twist angle/angle-of-attack ratio for minimum induced drag (elliptical spanwise loading)
η	= nondimensional semispan location (0 at root, 1 at tip)
Λ	= sweep angle at quarter chord, deg
Λ_β	= compressible sweep angle = arc tan[(tanΛ)/β], deg
λ	= taper ratio = tip chord/root chord

Introduction

The data presented in this note were generated as part of a large parametric study to determine basic aerodynamic characteristics over a wide subsonic Mach number range for uncambered, flat-plate wings having trapezoidal planforms (parallel root and tip chords) with straight leading and trailing edges. This type of wing was considered because of its desirability from a manufacturing standpoint. The amount of linear-lofted twist required for minimum induced drag for such wings was also determined. Body interference and viscous effects were not considered.

Discussion

The object of this note is to provide information that can be used to determine rapidly the complete wing characteristics for a near-optimum planform-twist combination at any specified subsonic flight condition. The lifting-surface computer program used in this study is described in Ref. 1. Briefly, the wing loading is numerically approxi-

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